

## Analyzing Liquidity and Absorption Limits of Electronic Markets with Volume Durations

Wing Lon Ng

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Analyzing Liquidity and Absorption Limits of  
Electronic Markets with Volume Durations

Wing Lon Ng <sup>1</sup>  
University of Muenster  
Institute of Econometrics and Economic Statistics  
Am Stadtgraben 9, 48143 Münster, Germany  
05wing@wiwi.uni-muenster.de

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### Abstract

This paper focuses on the liquidity of electronic stock markets applying a sequential estimation approach of models for volume durations with increasing threshold values. A modified ACD model with a Box-Tukey transformation and a flexible generalized Beta distribution is proposed to capture the changing cluster structure of duration processes. The estimation results with the German XETRA data reveal the market's absorption limit for high volumes of shares, expanding the time costs of illiquidity when trading these quantities.

**Key Words:** Ultra high frequency transaction data, limit order book, volume duration, autoregressive conditional duration, Box-Tukey transformation, temporal aggregation.

**JEL Classifications:** C22, C41, G14.

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1    **Introduction**

The literature on financial econometrics and quantitative finance has been used to focus on the stochastic process of daily prices or returns on assets and their volatility. Generally, most studies only pays little attention to other essential variables in financial markets and, thus, lose sight of the transaction volume – the second relevant factor when trading assets. It is often overlooked that the investigation of the trading quantity is also important, because it is one of the key factors responsible for market efficiency.

For decades, the volume was studied only in relationship with returns and volatility, serving as a explanatory variable for the analysis of the stochastic process. The literature goes back to Clark (1973), who introduced the use of subordinators in finance. In his model the daily price change represents a sum of within-day price changes. Since the trading volume is positively related to number of transactions, it helps explaining the variability of the price change. In another model proposed by Epps and Epps (1976) the change of the transaction price in the market is the mean of the changes of all trader’s reservation price, and they found a positive relationship between the absolute of the change in the market price and the trading volume. A generalization of these models are derived in Tauchen and Pitts (1983). Likewise, Gallant, Rossi, and Tauchen (1992) also found a positive correlation between volatility and volume, applying a semi-nonparametric method to estimate the joint density of price change and volume. A survey can be found in Karpoff (1987) and Jones, Kaul, and Lipson (1994). With the results of Anè and Geman (2000), the volume variable has become less important again (as in Jones, Kaul, and Lipson (1994)), because in their study with the substitution of subordinators with stochastic time changes, the number of trades represents a better stochastic clock than the volume for inducing normal returns.

In contrast to this wide stream of literature, this paper analyzes the time-varying liquidity of electronic markets by focusing on the relationship between time and volume. Liquidity has been approved as an important factor characterizing the market efficiency. The concept of liquidity has various definitions. Usually, it has three dimensions and is understood as the capability to trade (*a*) a large amount of shares (*b*) in a short time (*c*) at low cost. For example, Engle and Lange (2001) introduced the VNET measure that captures the *net* directional one-sided volume over the *price duration*, that is, the amount of excess volume that can be sustained before prices are adjusted. However, only regarding the absolute one-sided excess quantity, this approach does not take the total traded volume into account, which is essential when investigating the “entire” absorptive ability of the market. Hence, in this paper, a market is considered as liquid if unlimited volumes can be traded immediately, to exploit the overall absorption limits independent of the price impact. An adequate measurement is based on vol-

ume durations that considers the time and volume dimension of the trading process. It is defined as the time elapsed until a certain quantity of shares is absorbed by the market (Bauwens and Giot (2001)) and, thus, provide a good indicator for the time costs of (il)liquidity (Hautsch (2004)).

The analysis of volume durations is important especially for liquidity traders. Short volume durations imply that threshold amount of stocks can be traded very quickly, whereas long ones signalize a (temporarily) illiquid market. Since the empirical cluster structures of volume durations indicate different degrees of market liquidity, they become important decision factors influencing traders' order-placement strategies. Thus, modeling volume durations can help finding the absorption limits of the market, saving the (time) costs of liquidity for traders.

Therefore, the main objective of this paper is to study the dynamics of volume durations by applying the ACD framework, an appropriate econometric tool for analyzing financial duration data (for a survey, see Bauwens, Giot, Grammig, and Veredas (2004) or Engle and Russell (2004)). With the increased availability of ultra-high-frequency transaction data in the last few years, many researchers have pushed the further development of the ACD model in order to describe limit order book activities more accurately. Recently, Fernandes and Grammig (2006) have introduced an augmented version that encompasses all specifications discussed in the literature by adopting the results of Hentschel (1995) into the ACD methodology. However, since the asymmetry effects of volume durations are assumed to be marginal, this paper concentrates on a Box-Tukey specification that is parsimonious and flexible as well (Box and Cox (1964)). Similar to Fernandes and Grammig (2006), the modified ACD model with this transformation is also able to induce various non-monotonic shapes of the news-impact curve to tailor the data more precisely. Interestingly, since the issue addressed here concerns the market's liquidity given an predetermined cumulated volume, the shifting parameters in the Box-Tukey specification can be used to identify the absorption limit.

Furthermore, a generalized Beta distribution is proposed that nests more than 30 distributions as limiting or special cases, including those already discussed in the ACD literature. It can be shown that the generalized Beta distribution allows for hazard functions with diverse slopes, implying different duration dependence of the data. As extension to several studies in this research field (see, for example, Hautsch (2002) and Hautsch (2003)), this paper investigates the entire evolution of volume duration processes by successively increasing the threshold value, resulting in a sequential estimation approach considering "temporal" aggregation effects.

The paper is structured as follows: In Section 2, the Box-Tukey-ACD model and concept of volume durations will be introduced. Section 3 describes the smoothing technique for deseasonalization and deals with the ML estimation. In Section 4, the data and results are presented. Section 5

concludes.

## 2 Methodology

Since ultra-high-frequency transaction data arrive at irregular time intervals, researchers must not only consider the key variables themselves (price, return, volume, etc.), but also their stochastic arrival times (Hafner (2005)). Hence, a duration analysis is the common approach to describe these time stamped data. Let  $X_i = t_i - t_{i-1}$  (with  $t_0 = 0$ ), where  $X_i$  is the  $i^{th}$  trade duration between the  $i^{th}$  and  $(i-1)^{th}$  transaction. Further, define  $\Psi_i \equiv E(X_i | \mathcal{F}_{i-1})$ , that is, the conditional expectation of the duration, given  $\mathcal{F}_{i-1}$  i.e. the information available at  $t_{i-1}$ . The common ACD framework, a popular tool in recent financial econometrics, models a dynamic point process in which the conditional expectation is written as a linear function of past durations

$$\Psi_i = \omega + \sum_{j=1}^p \alpha_j X_{i-j} + \sum_{k=1}^q \beta_k \Psi_{i-k} \quad (1)$$

(see Engle and Russell (1998) and Engle (2000)). It is assumed that

$$X_i = \Psi_i \cdot \varepsilon_i \quad (2)$$

with (i.i.d.-)innovations  $\varepsilon_i$ . The  $\varepsilon_i \equiv \frac{X_i}{\Psi_i}$  are also called standardized durations. Their density function  $f(\varepsilon_i)$  is parametrized with (a) normalization  $E(\varepsilon_i) = 1$  by construction, and (b) a non-negative support to avoid negative durations. Since the assumption of linearity is often too restrictive to capture the duration process, several models have been developed and modified with other dependence structures of the conditional mean in order to account for nonlinear impacts. Generally, new and different types of ACD models can be created by varying the functional form of  $\Psi_i$  in the model's mean equation.

Based on the Box-Cox-ACD, first introduced by Dufour and Engle (2000), modified and extended by Hautsch (2002), a Box-Tukey transformation of the durations is proposed in this study

$$\underbrace{\frac{(\Psi_i + \eta_1)^{\kappa_1} - 1}{\kappa_1}}_{\Psi_i^{BT}} = \sum_{j=1}^p \alpha_j \underbrace{\left( \frac{(\varepsilon_{i-j} + \eta_2)^{\kappa_2} - 1}{\kappa_2} \right)}_{\varepsilon_{i-j}^{BT}} + \sum_{k=1}^q \beta_k \underbrace{\left( \frac{(\Psi_{i-k} + \eta_1)^{\kappa_1} - 1}{\kappa_1} \right)}_{\Psi_{i-k}^{BT}} \quad (3)$$

Here,  $\Psi_i^{BT}$  and  $\varepsilon_i^{BT}$  are the Box-Tukey-transformed durations. This extended Box-Cox offers an extra degree of flexibility to tailor the data, although the parametrization remains parsimonious. Instead of the common intercept  $\omega$ , two new shifting parameters  $\eta_1$  and  $\eta_2$  are introduced in order

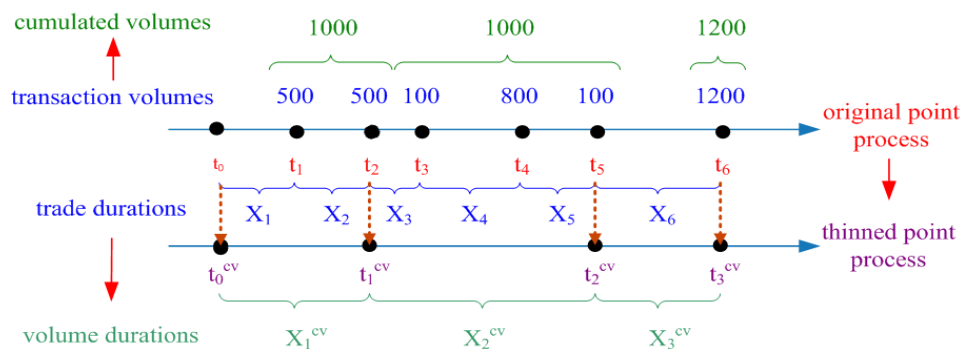


Figure 1: Generating volume durations with  $cv = 1000$ .

to allow a more flexible adjustment of the news-impact curve which measures the influence of innovations on the conditional mean. In contrast to the basic ACD model, it allows additive innovation shocks with  $\kappa_1 = \kappa_2 = 1$ , as well as nonlinear ones for  $\kappa_1, \kappa_2 \neq 1$ . Due to the double-shifting of durations in the conditional mean equation, it has similar statistical properties to more sophisticated ACD models focusing on the kink in the innovations' news-impact curves, such as the augmented ACD model of Fernandes and Grammig (2006) (for stationarity conditions, see also Carrasco and Chen (2002) and Meitz and Saikkonen (2005)).

Volume durations are defined as the period elapsed until a given amount of shares,  $cv$  (cumulated volume), is traded on the market (Bauwens and Giot (2001)). In contrast to trade durations that stand for the time elapsed between two consecutive transactions, volume durations correspond to the time interval required to observe a certain cumulation of traded shares. In order to model the stochastic process of volume durations, the ACD framework can be simply adapted to the new thinned point process  $(t_j^{cv})_{j \in \mathbb{N}}$ , where  $t_j^{cv}$  now denotes the points each time the market has absorbed (at least) the pre-determined volume  $cv$ . As discernible in Figure 1, the resulting volume duration process  $(X_j^{cv})_{j \in \mathbb{N}}$  with  $X_j^{cv} = t_j^{cv} - t_{j-1}^{cv}$  is generated by cumulating single trade durations until the sum of single transaction volumes  $vol_i$  has been achieved or exceeded  $cv$ , i.e.  $\sum vol_{t_i} \geq cv, \forall t_i \in (t_{j-1}^{cv}; t_j^{cv}]$ . Subsequently, the model's new filtration is the modified  $\sigma$ -field  $\mathcal{F}_j^{cv} = \sigma(t_j^{cv}, t_{j-1}^{cv}, \dots, t_1^{cv}, t_0^{cv})$ .

Note that the single transaction volume  $vol_i$  does not represent the relevant random variable, but the duration required to absorb the predetermined cumulated volume  $cv$ . In contrast to the literature, where the duration process is analyzed for only one or a few "suitable" threshold values (see, for example, Bauwens and Giot (2001), Hautsch (2002) or Fernandes and Grammig (2006)), this paper focuses on the development of the models'



parameters by successively increasing  $cv_k$  representing  $k \cdot 500$  units. In the following, a sequence of 100 model estimations of successively aggregated sub-datasets is run with  $k = 1, 2, \dots, 100$ . The first model concentrates on the points signaling that the market has absorbed  $cv_1$ , i.e.  $1 \cdot 500$  shares, the second one analyzes the duration of trading  $cv_2 \hat{=} 2 \cdot 500$  shares, etc. To keep the computational burdens of the 100 estimation procedures within manageable limits and in order to achieve a more reliable comparison of the different model's parameters, this paper concentrates only on a *Box-Tukey-ACD(1,1)* specification.

Taking temporal aggregation effects into account, the formulas of Drost and Nijman (1993) for computing low-frequency GARCH models were applied to get better start values for the estimation (see also Drost and Werker (1996) and Ghysels and Jasiak (1998)). Due to the irregular spacing in time and the existence of extreme long durations for high  $cv$  values, the "frequency parameter"  $m$  (originally for equidistant data) in their model was replaced by

$$m_k = \frac{\frac{1}{n_k} \sum_{i=1}^{n_k} X_i^{cv_k}}{\frac{1}{n_1} \sum_{i=1}^{n_1} X_i^{cv_1}}$$

to consider the non-constant temporal aggregation of the duration models.  $n_k$  stands for the number of observations in the  $k$ -th sub-dataset. Having the estimated parameters for the first aggregation level, all following start values for the  $\alpha$  and  $\beta$  parameter of the remaining 99 models are obtained by

$$\alpha_{(m_k)} = (\alpha + \beta)^{m_k} - \beta_{(m_k)}$$

and

$$\beta_{(m_k)} = \frac{1}{2const} \pm \sqrt{\frac{1}{4const^2} - 1}$$

with

$$const = \frac{\beta (\alpha + \beta)^{m_k-1}}{1 + \alpha^2 \frac{1-(\alpha+\beta)^{2m_k-2}}{1-(\alpha+\beta)^2} + \beta^2 (\beta + a)^{2m_k-2}}$$

and  $\beta_{(m_k)} \in (0; 1)$ . The start values for the Box-Cox and the Box-Tukey parameters are reset to  $\kappa = 0.99999$  and  $\eta = 0.00001$  for each estimation, beginning from a quasi-linear specification of the mean equation.

### 3 Estimation and Diagnostics

It is well-known that intraday data have a consistent diurnal pattern of trading activities over the course of a trading day, due to certain institutional characteristics of organized financial markets, such as opening and closing hours or intraday auctions. Since it is necessary to take the daily deterministic seasonality into account, smoothing techniques are required

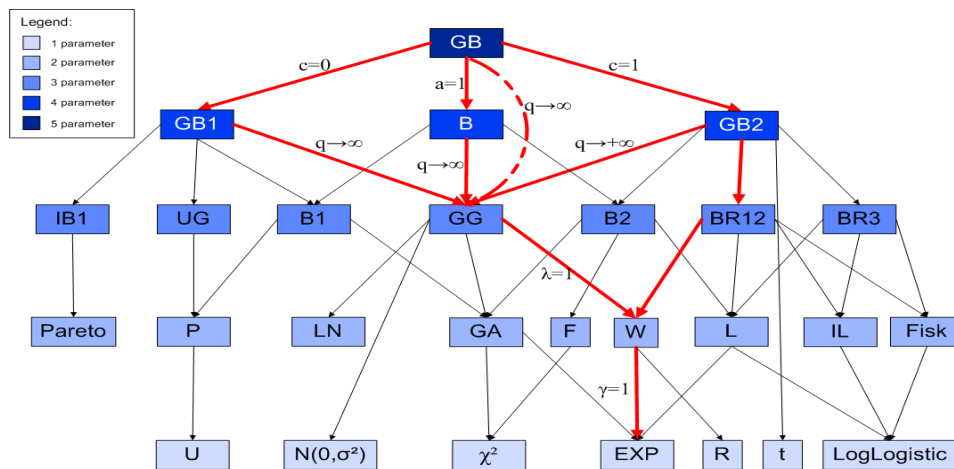


Figure 2: Distribution tree of McDonald and Xu (1995), p. 139.

to get deseasonalized observations. Let  $\tilde{X}_i$  denote the observed duration. Instead of applying cubic splines, where the positions of the nodes have to be carefully set, a kernel regression with an cross-validated bandwidth  $h_c$  was performed. Using the Gaussian density as kernel function  $K(\cdot)$ , the diurnal periodic component (dependent on daytime  $d$ ) can be computed by the Nadaraya-Watson estimator

$$NW(d_i) = \frac{\sum_{i=1}^n \tilde{X}_i \cdot K\left(\frac{d-d_i}{h_c}\right)}{\sum_{i=1}^n K\left(\frac{d-d_i}{h_c}\right)}. \quad (4)$$

Thus,  $X_i \equiv \frac{\tilde{X}_i}{NW(d_i)}$  is the deseasonalized duration and should have no diurnal pattern and a unit mean.

The ACD model is estimated by maximum-likelihood. Originally, Engle and Russell (1997) used an Exponential ( $EXP$ ) and a Weibull ( $W$ ) distribution for the residuals (see Figure 2), whereas other authors favoured more flexible alternatives (see, for example Bauwens, Giot, Grammig, and Veredas (2004)). In this study, the density of the innovations  $f(\cdot)$  is modelled by the generalized Beta ( $GB$ ) distribution proposed by McDonald and Xu (1995)

$$f(\varepsilon_i; a, b, c, p, q) = \left( \frac{|a| \cdot \Gamma(p+q)}{\Gamma(p)\Gamma(q)} \right) \frac{\varepsilon_i^{ap-1} (1 - (1-c)(\frac{\varepsilon_i}{b})^a)^{q-1}}{b^{ap} (1 + c(\frac{\varepsilon_i}{b})^a)^{p+q}}$$

with the parameter  $a$  controlling the peakedness of the density, the scale parameter  $b$ , the smoothing parameter  $c$  and the parameters  $p$  and  $q$  influencing the shape and the skewness (whereby  $a, b, p, q > 0$  and  $0 \leq c \leq 1$ ; for statistical properties, estimation and the calculation of the moments of  $\varepsilon_i$ ,

see also Kleiber and Kotz (2003)). Due to its flexibility the generalized Beta distribution nests all distributions already discussed in the ACD-literature as limiting or special cases, such as the Burr-distributions type *III* and type *XII* (BR12, see Fernandes and Grammig (2006)), the generalized Gamma (*GG*, see Lunde (2000)) or the generalized Beta of the second kind (*GB2*). A variation of this distribution is also called “generalized F-distribution” (see Hautsch (2004) and Kalbfleisch and Prentice (2002)). The moments of  $\varepsilon_i$  can be obtained by

$$E(\varepsilon_i^m) = b^m \frac{B(p + \frac{m}{a}, q)}{B(p, q)} \cdot \left( {}_2F_1 \left[ \begin{matrix} p + \frac{m}{a}, \frac{m}{a} \\ p + \frac{m}{a} + q \end{matrix} ; c \right] \right)$$

with Beta function  $B(\cdot)$  and the Gaussian hypergeometric function

$${}_2F_1 \left[ \begin{matrix} u_1 & u_2 \\ v_1 \end{matrix} ; z \right] = \sum_{n=0}^{\infty} \frac{(u_1)_n (u_2)_n}{(v_1)_n} \frac{z^n}{n!} \quad .$$

Hence, the log-likelihood function of the model with a generalized Beta distribution for the innovations is

$$\begin{aligned} \mathcal{L}(x_1, \dots, x_n) = & n \ln \left( \frac{|a| \cdot \Gamma(p+q)}{\Gamma(p) \Gamma(q)} \right) - apn \ln(b) \\ & - (ap-1) \sum_{i=1}^n \ln(x_i) - ap \sum_{i=1}^n \ln(\Psi_i) \\ & + (q-1) \sum_{i=1}^n \ln \left( 1 - (1-c) \left( \frac{x_i}{b\Psi_i} \right)^a \right) \\ & - (p+q) \sum_{i=1}^n \ln \left( 1 + c \left( \frac{x_i}{b\Psi_i} \right)^a \right) \end{aligned}$$

with

$$b = \frac{\Gamma(p + \frac{1}{a} + q) \Gamma(p)}{\Gamma(p + \frac{1}{a}) \Gamma(p+q)} \cdot \left( {}_2F_1 \left[ \begin{matrix} p + \frac{1}{a}, \frac{1}{a} \\ p + \frac{1}{a} + q \end{matrix} ; c \right] \right)^{-1} ,$$

yielding unit expectation as required.

To check all 100 ACD model’s diagnostics, three tests are applied: First of all, one can examine the properties of the residuals such as their autocorrelation structure which is i.i.d. under correct model specification. Hence, a Ljung-Box-test with modified Portmanteau statistic proposed by Li and Li (2005) is performed for the lag orders 5 and 10. Secondly, another general test is based on the integration of intensity  $h(\cdot)$  over duration  $(t_i - t_{i-1})$

$$\Lambda_i = \int_{s=t_{i-1}}^{t_i} h(s|\mathcal{F}_s) ds \quad (5)$$

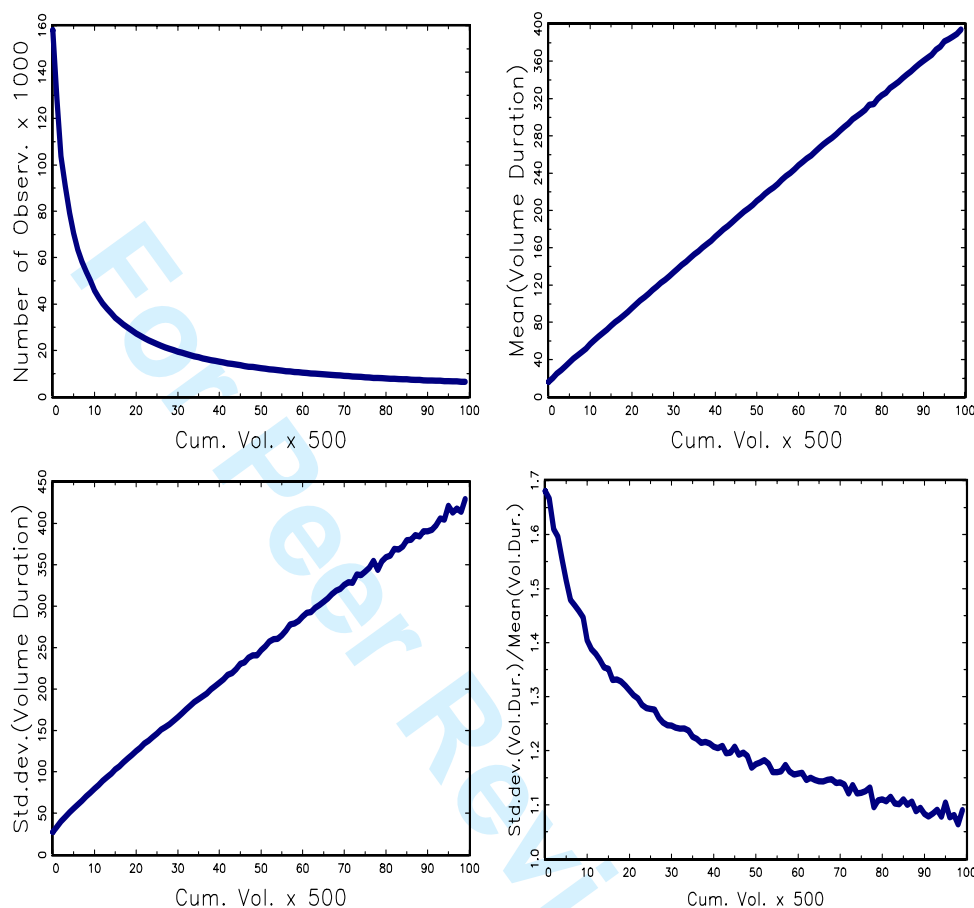


Figure 3: Description of volume durations (in sec.) of all 100 aggregated sub-datasets.

that follows a standard exponential distribution under the correct model specification, as illustrated in Russell (1999). Thus, Engle and Russell (2004) suggested testing the overdispersion of  $\hat{\Lambda}_i$  (henceforth ER-Test). Finally, the theoretical distribution of the residuals implied by the estimated parameters  $f(\varepsilon, \hat{\theta})$  can be compared with the empirical one  $\hat{f}(\hat{\varepsilon})$ . For this purpose, Fernandes and Grammig (2006) developed the D-test(-statistic) to quantify the difference between them, which should be zero under correct model specification.

## 4 Dataset and Empirical Results

The transaction data of the Deutsche Telekom stock was extracted from the open order book of the German XETRA system, which is an order-

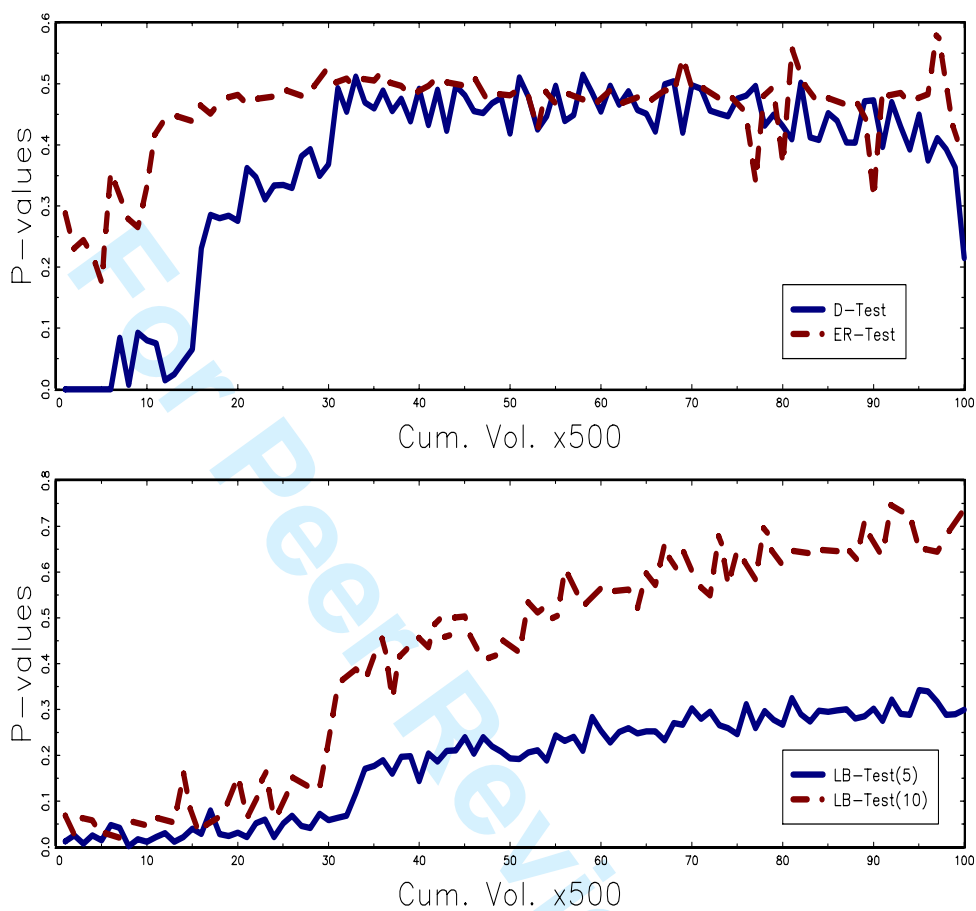


Figure 4: P-values of the Ljung-Box test, the D-test and the ER-test for all 100 ACD models

driven market. The sample includes 225905 single transactions from 3<sup>rd</sup> July until 6<sup>th</sup> October 2000, observed for 69 trading days over 14 weeks. The continuous trading phase starts after the opening auction at 9 a.m. and ends before the closing auction at 8 p.m. Further, it is interrupted by (at least) two intraday auctions at 1 p.m. and 5 p.m., each lasting at most 120 seconds. The electronic trading is based on an automatic matching algorithm, generally following a strict price-time priority of orders.

Figure 3 describes the volume durations data and shows how the mean and the standard deviation vary according to the threshold values. As expected, these measures increase, whereas the number of observations decreases in respect to a rising aggregation level. In contrast to the findings of Bauwens and Giot (2001) and Hautsch (2004), the dispersion (defined in duration analysis as standard deviation divided by mean) declines as well

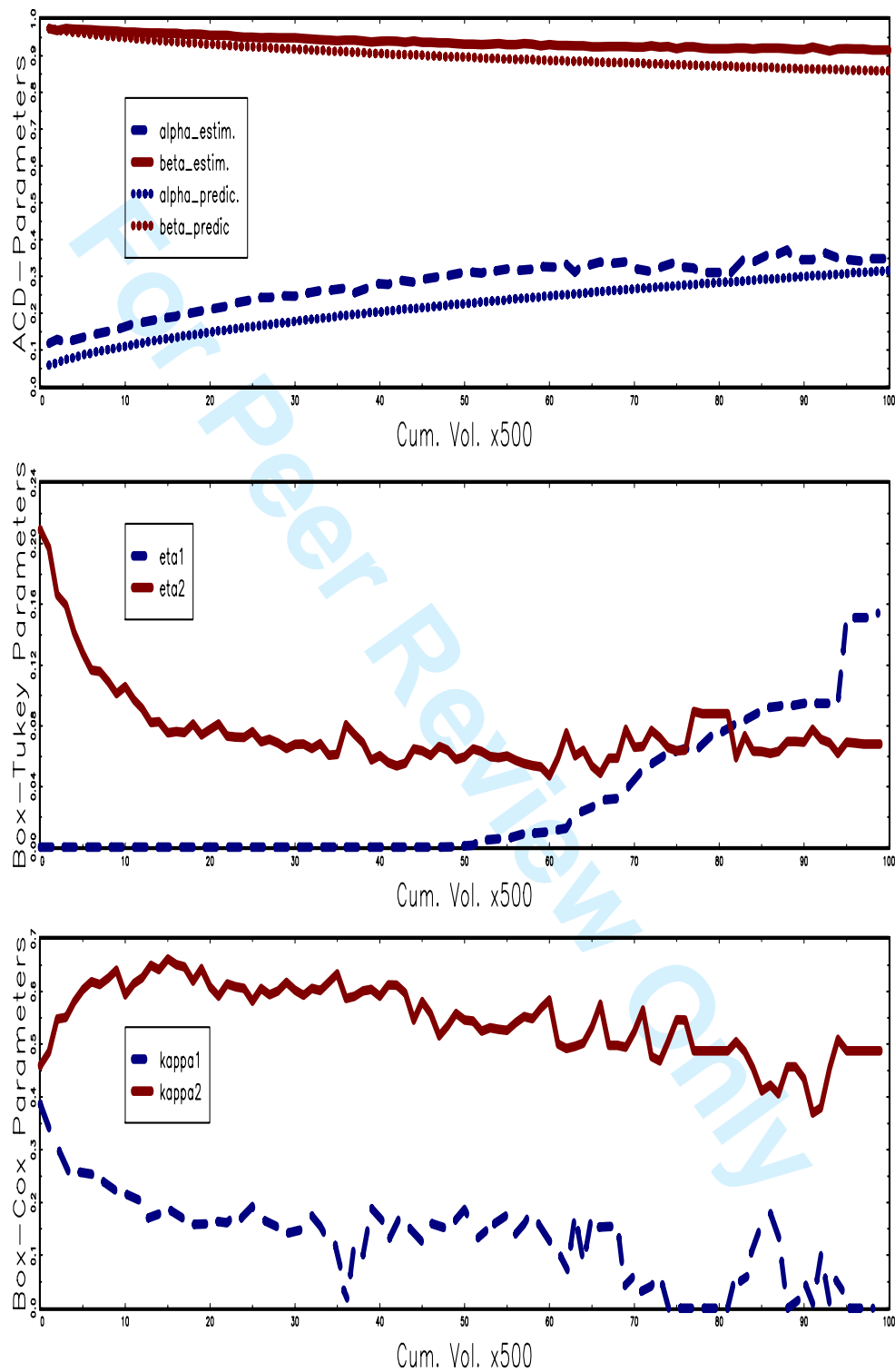


Figure 5: Estimated parameters of the ACD models for all 100 sub-datasets.

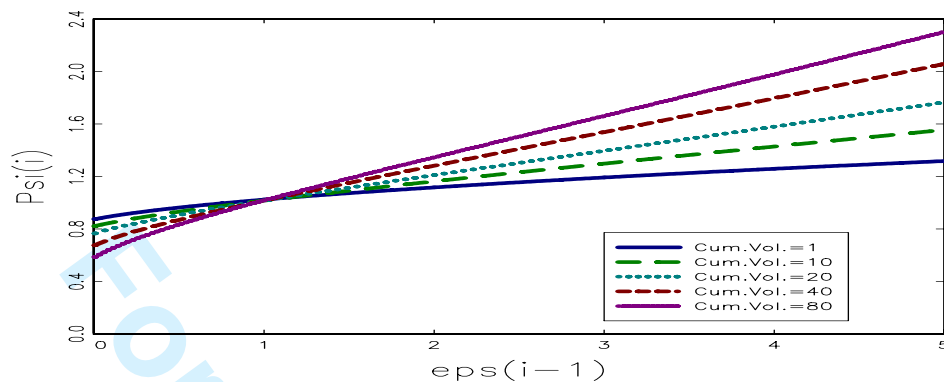


Figure 6: News-impact curve for different threshold values

but still indicates overdispersion of the volume durations for high threshold values. Since the standard deviation always exceeds the mean, it implies a negative duration dependence, that is, the waiting time to absorb  $cv$  is less likely to end the longer it lasts (Winkelmann (1998)).

First of all, Figure 4 shows the  $p$ -values of the D-Test, the ER-Test and the Ljung-Box test (with the modified portmanteau statistic of Li and Li (2005)) of all estimated ACD models. The estimates and their standard errors (in parenthesis) of four selected duration models are reported in Table 1. Except the test of overdispersion by Engle and Russell (2004), all other tests reject a correct specification of the models for small aggregation levels. The D-test rejects the null for  $cv \leq 15$  and the Ljung-Box tests reject for  $cv \leq 33$ , indicating that the model of lag orders (1, 1) with the Box-Tukey transformation is not able to capture the dynamics of volume durations in the lower threshold levels. In contrast, the same model specification seem to show a good performance for all higher  $cv$  values, similar to the findings of Hautsch (2004). Hence, only estimated models with  $cv > 34$  are considered in the following.

In general, the ACD models detect a strong cluster structure of volume durations, signaling a certain behavioral pattern of traders (see Figure 5). Long volume durations tend to be followed by long ones and short durations by short ones,  $\hat{\alpha}_1, \hat{\beta}_1 > 0$ , revealing a serial dependence. These findings are in line the common hypothesis of information-based market microstructure theory, where the trading and ordering process represents a source of information (for overview, see O'Hara (1997)). According to the theory, long durations suggests that uninformed traders (still) believe that the underlying value of the asset has not changed and only trade because of their own portfolio optimization. Contrary, short durations and, hence, intensive trading signalize the presence of informed traders who are assumed to make money by capitalizing on their informational advantage. The more

Parameters		$cv = 1$	$cv = 20$	$cv = 40$	$cv = 80$
Box-Tukey-ACD model					
$\alpha_1$	last standardized duration	0.1181 (0.0015)	0.2083 (0.0063)	0.2811 (0.0096)	0.3094 (0.0165)
$\beta_1$	last conditional duration	0.9740 (0.0009)	0.9557 (0.0028)	0.9382 (0.0045)	0.9196 (0.0058)
$\eta_1$	Box-Tukey <sub>1</sub>	0.0000 (.)	0.0000 (.)	0.0000 (.)	0.0793 (0.0105)
$\eta_2$	Box-Tukey <sub>2</sub>	0.2093 (0.0044)	0.0737 (0.0104)	0.0573 (0.0106)	0.0879 (0.0009)
$\kappa_1$	Box-Cox <sub>1</sub>	0.3884 (0.0210)	0.1720 (0.0364)	0.1893 (0.0513)	0.0000 (.)
$\kappa_2$	Box-Cox <sub>2</sub>	0.4588 (0.0099)	0.6436 (0.0355)	0.6041 (0.0476)	0.4871 (0.0200)
Gen. Beta distribution					
$a$	peakedness of GBeta distr.	0.5972 (0.0055)	1.6981 (0.0912)	2.0549 (0.1280)	1.9258 (0.1365)
$p$	shape of GBeta distr.	2.0795 (0.0380)	0.8189 (0.0567)	0.8761 (0.0716)	1.2975 (0.1258)
$q$	shape of GBeta distr.	65.7257 (2.3774)	3.4026 (0.4695)	2.8113 (0.4024)	3.8918 (0.6904)
$c$	smooth of GBeta distr.	0.8537 (0.0282)	0.8651 (0.0419)	0.9562 (0.0825)	0.8602 (0.0913)

Table 1: Estimation results of the Box-Tukey-ACD model for selected  $cv$ -values

information that is available in the market, the faster they have to react.

In this study, the Box-Tukey-ACD empirically converges to either the Box-Cox-ACD or the Log2-ACD specification, where in both cases stationarity is ensured by  $|\hat{\beta}_1| < 1$ , as discussed in Fernandes and Grammig (2006). As evident in the upper panel of Figure 5, the persistence of the process is strong and diminishes slightly (from  $\hat{\beta}_1^{(cv=34)} = 0.9427$  to  $\hat{\beta}_1^{(cv=100)} = 0.9155$ ), whereas  $\hat{\alpha}_1$  increases for a growing  $cv$  (from  $\hat{\alpha}_1^{(cv=34)} = 0.2606$  to  $\hat{\alpha}_1^{(cv=100)} = 0.3481$ ). Economically speaking, there is no (more) need for a rapid speed of updating the information in the market when regarding orders with high volumes, which necessarily take a long time to be absorbed by the market. According to the results of Easley and O'Hara (1992), the information efficiency of the market moves from a semi-strong to a strong level. Interestingly, the predicted  $\alpha$  and  $\beta$  parameters by the adjusted formula by Drost and Nijman (1993) that were used as start values for the



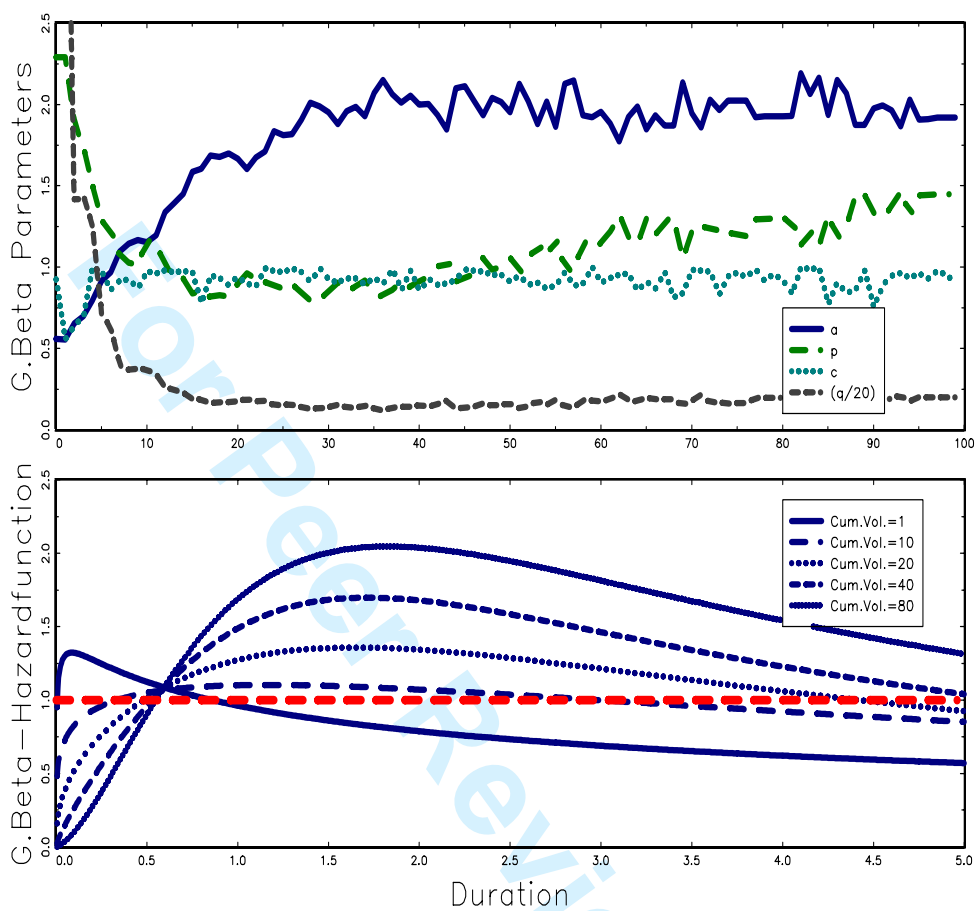


Figure 7: Estimated parameters of the generalized Beta distribution of all 100 ACD models and the implied hazard functions for different threshold values.

estimation procedure seem to serve as a lower bound.

Since both Box-Cox parameters  $\hat{\kappa}_1, \hat{\kappa}_2 < 1$  (see lower panel of Figure 5), the dependence structure is nonlinear, implying concave news-impact curves, similar to the results of Fernandes and Grammig (2006). This means that the conditional duration must be adjusted more extensively during hectic periods (short durations) than in calm ones (long durations). The parameter  $\hat{\kappa}_2$  is between 0.35 and 0.65, whereas  $\hat{\kappa}_1$  is slightly decreasing, going to zero and, thus, converging to the so-called Log-ACD model for a few high  $cv$  values. Furthermore, both Box-Tukey parameters  $\hat{\eta}_1, \hat{\eta}_2 \geq 0$  (see middle panel of Figure 5), which indicates that the durations must be shifted in the overall adjustment process. As expected, the standardized duration  $\varepsilon_i$  always has to be enlarged with  $\hat{\eta}_2$ , whereas the conditional mean duration

$\Psi$  only needs to be shifted with  $\hat{\eta}_1$  when regarding a high  $cv$ . In these cases, the model requires an additional intercept in order to capture the dynamics adequately. The Box-Tukey parameter  $\hat{\eta}_1$  implies that the market is not liquid enough to absorb large amounts of shares ( $cv \geq 50$ ) in a relatively short time, thus inducing additional time costs of liquidity.

In respect of reducing these costs, the results suggest that traders should either stop submitting new orders when observing these large amounts of traded shares or not exceed these thresholds values in case of fast (liquidity) block trading. A comparison of different news-impact curves for different threshold values is shown in Figure 6, displaying the market's reaction time. The influence of a high (low) duration shock  $\varepsilon_{i-1}$  will be the greater (smaller), the higher  $cv$ .

The errors  $\hat{\varepsilon}_i = x_i / \hat{\Psi}_i$  are generalized Beta distributed. The development of the distribution parameters  $\hat{a}, \hat{p}, \hat{q}$  and  $\hat{c}$  are depicted in the upper panel of Figure 7, different implied hazard functions at the respective threshold levels are shown in the lower panel. As discernible, all hazard functions have a similar form and follow a non-monotonous concave slope, where the position of the maximum of the curve – determining the switch of the duration dependence – is closer to zero the smaller the threshold value. For example, the hazard function of the residuals for  $cv = 1$  immediately decreases after a short jump, whereas other curves first show a moderate rise and then decline. This indicates that small quantities are most probably to be absorbed “immediately”, while large amounts are more likely to be traded, the longer the time that has elapsed. In general, the hazard functions only differ in the time, where the positive duration ends and the negative begins, signalling that the spell to absorb  $cv$  is less likely to end the longer it lasts.

## 5 Conclusion

This paper investigates the time varying liquidity of limit order books by analyzing the dynamics of volume durations with increasing threshold values. The main objective was to study the development of the models' parameters in order to reveal the market's absorption speed at different aggregation levels. Applying a flexible generalized Beta distribution to capture the extreme durations, and a Box-Tukey-transformation to account for a nonlinear news-impact curve, a sequence of *ACD* models was estimated to examine how the time series properties change as the threshold number of shares traded rises. Using detailed transaction data from the German XETRA system, the empirical results show that the cluster structure of the process and the market's absorption speed decreases continuously for highly cumulated volumes. The Box-Tukey parameter signalize that the market is not liquid enough to absorb large amounts of shares in a relatively short time, thus inducing additional time costs of liquidity.

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